Accessible Time Interval Based Local Positioning System: Applications for Self-Driving Cars in Smart Cities

Jihene Rezgui, Antoine Bolduc, Enric Soldevila, Yousra Azmour, Fadner Dieujuste Richardson
Laboratoire **R**echerche **I**nformatique **M**aisonneuve (LRIMa)
Montreal, Canada
jrezgui@cmaisonneuve.qc.ca

Abstract – **The LRIMa City testbed is a comprehensive expandable, and versatile smart city framework designed to advance research and learning in the fields of Internet of Things. In this paper, we propose an accessible yet accurate Local Positioning System (LPS) leveraging ultrasonic signals to determine vehicle locations by using a network of towers equipped with ultrasonic receivers placed at the corners of the city. Each vehicle is equipped with an antenna that emits ultrasound waves in all directions, which are then detected by the towers. Then, we use the time differences between signal receptions, also known as difference time of arrival (DTOA), to triangulate the vehicle's position. Through preliminary simulations and testing, we estimate that our LPS system is a promising solution with a location accuracy of 2.6 cm. Moreover, we developed a City Centralized Web Interface, named CCWI, to enable seamless control and monitoring of fog computing on-device solutions. Finally, our proposed LPS system is to be directly connected to the CCWI to allow vehicle localization at a small scale within the LRIMa Smart City testbed.**

Keywords: Internet of Things, Smart City, LPS, Aliot, CCWI.

I. INTRODUCTION

In our ongoing quest to develop and refine smart city technologies, we have made significant advancements since our previous work on LRIMa's Smart City [1], a testbed for exploring IoT architectures, including cloud centralized and fog on-device computation [2]. Our initial efforts addressed issues such as high latency and faulty navigation in cloud-based solutions. Building on this foundation, we now introduce an accessible Local Positioning System (LPS) at low cost and lesser complexity. LPSs are already well documented, with papers covering various methods such as ultrasonic systems combining time-of-flight and phase-shift methods [3-4] or Indoor Positioning Systems (IPS) [5-6]. Nevertheless, possibilities to leverage and experiment with this system are not the most accessible, sometimes requiring costly components or advanced knowledge in electrical engineering. Our cost efficient and accessible LPS works with our new motorized vehicles equipped with antennas emitting ultrasound waves in all directions. These signals then get picked up by four towers located in each corner of the Smart City. Finally, we use the time differences between signal receptions, also known as Difference Time of Arrival (DTOA) to triangulate the vehicle's position using a clever mathematical principle, hyperbolas. Moreover, using our previous work Aliot, an IoT Development Kit for Personalized Smart Ecosystems [7], the new LRIMa Smart City includes the City Centralized Web Interface (CCWI), that allows for real-time monitoring of vehicles' activity. Finally, we wrote a comprehensive conception guide for the smart cars allowing their reusability for further research. These additions continue to evolve LRIMa's city as a cutting-edge testbed for smart city technologies, paving the way for more intelligent, responsive, and efficient urban environments.

Our contributions in this paper can be summarized as follows: **(1)** We have designed an accessible Local Positioning System (LPS) to accurately locate a motorized vehicle within the city, using ultrasonic signals and a triangulation method inspired by hyperbolas; **(2)** We developed a City Centralized Web Interface (CCWI) in order to visualize the LPS in action and to control and monitor the LRIMa City; **(3)** We created new modular and reusable vehicles allowing LPS integration; **(4)** We wrote a comprehensive guide on creating these vehicles, fostering innovation and practical learning and **(5)** We showed the deviation of our LPS solution compared to manual measuring is within ±2.6cm, representing a very accurate positioning.

***Outline**:* Section II gives a brief overview of related works in LPS and smart city projects. Section III explains how to accurately locate a Car in the Smart City using hyperbolas. Section IV presents the CCWI for the city and our modular and reusable cars. Section V shows our results. Finally, section VI concludes the paper.

II. RELATED WORK

A. Previous LPS and triangulation methods

Ultrasonic transmission and reception technology has been used for a wide variety of robotic related applications for several decades, such as distance measurement [3-4], navigation [5] and indoor positioning systems (IPS) [6]. Additionally, ultrasonic sensors have been widely used for robotics applications in Local Positioning Systems (LPS), in order to track small robots at relatively high accuracy. Such systems often rely on an array of transmitters sending ultrasounds to receivers mounted on the robot as well as an analog decoding of the signal received. While such systems work quite well and offer high accuracy positioning results, they often require more complex and expensive materials, they necessitate extensive knowledge in frequency analysis, and they can be a computationally heavy undertaking for a robot that is already required to perform complex operations. While still maintaining high positioning accuracy, the proposed LPS is specifically aimed towards beginner robotics enthusiasts as the theoretical knowledge required to understand the system's functionalities is very accessible and further explained in detail. While most other systems rely upon analysing complex analog ultrasonic signals, the proposed system simply relies on digital ultrasonic detection from the widely used HC-SR04 [8]. Furthermore, the proposed mathematical principle required to triangulate the position of a DTOA based system, the hyperbola, is quite intuitive and accessible for all. While hyperbola triangulation is regularly used for calculating the position of an object [9-10], most systems use transmitters as the fixed reference points, whereas our LPS reduces the Smart Car's computational requirements by placing the receivers on the four towers cornering the city and the transmitters on the car.

B. Similar integrated Smart Cities

Multiple research projects have introduced the idea of a smart city and its aspects in both research and education, such as Micro:bit [11] and STEAM [12]. Notably, DuckieTown [13] is interesting as it aligns our research's goals due to its similar focus on simulating a smart city environment with limited resources. This comparison aims to highlight the innovative approaches brought by LRIMa City.

While DuckieTown focuses on simple, accessible learning for autonomous vehicle concepts, LRIMa City steps up the technological complexity and incorporates more elements of a smart city, such as LPS, functioning traffic lights, smart bridges, etc. The addition of the City Centralized Web Interface (CCWI) in LRIMa City enhances real-time monitoring and control capabilities, making it a more comprehensive and advanced testing ground for self-driving vehicle technologies and smart city innovations. This makes LRIMa City not only a robust research platform but also a valuable educational tool for hands-on learning and practical application of AI and IoT technologies.

III. OUR PROPOSED LOCAL POSITIONING SYSTEM

A Local Positioning System (LPS) would allow for the localization of a vehicle within a city and the transmission of its coordinates to a graphical interface to accurately and precisely illustrate its position. The core idea behind this LPS is that the Smart vehicle is equipped with an antenna-like module that emits ultrasonic signals in all directions that are to be received by four towers equipped with ultrasonic receivers located at the corners of the city as shown in **Fig.1**. **But how can we calculate the vehicle's position based on these towers?**

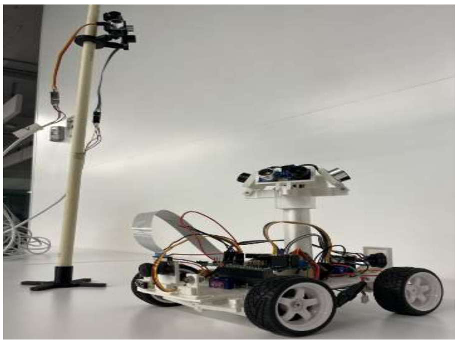

Fig.1. LRIMa's Smart Car equipped by the antenna-like ultrasonic transmitter module and a receiving tower to be located at the corner of the Smart City.

Fortunately, sound travels at a relatively slow speed, which means each tower will receive the same ultrasonic signal with a delay of several microseconds depending on the vehicle's proximity to each tower. The time difference between the receipt of these signals can be easily converted into distance differences between the towers (by multiplying the time difference by the speed of sound, which is 343 m/s in air at 20°C). **But how can we determine the vehicle's exact position if we only know these relative distances?**

The initial breakthrough came with the discovery of a similar question on Mathematics StackExchange answered by Eric Towers [14]. User Jim McGaw sought to understand the set of points representing the possible location of a sound-emitting object based on the time interval between sound detection by two audio receivers. Towers then explains the concept of hyperbolas and applies it to the situation of the sound-emitting object. Following are the explanations and demonstrations of the various mathematical principles that make up the design of our proposed LPS.

A. The Hyperbola

The hyperbola is a geometric plane curve that possesses several interesting characteristics that apply perfectly in the context of the triangulation we are aiming to perform.

Any point (x, y) on the hyperbola satisfies the following formula Eq.1, given that c is the distance between the origin and a focal point, and a is the distance between the origin and the vertex of the hyperbola.

$$\sqrt{(x-c)^2+y^2} - \sqrt{(x+c)^2+y^2} = \pm 2a$$

Eq.1. The hyperbola

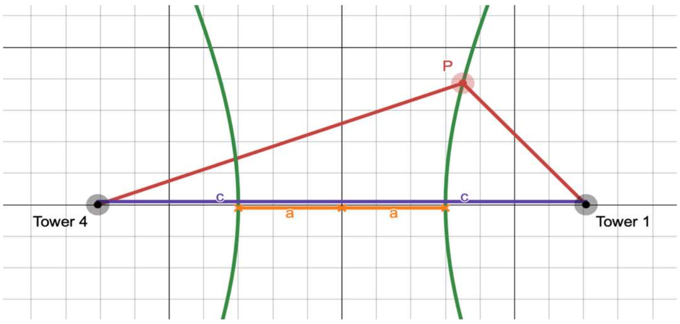

Fig.2. Visualisation of the use of a hyperbola in the context of locating a Smart Car within the Smart City, as well as point P, which could be a possible location given only that the Car is 60cm closer to tower 1 than tower 4.

In the context of triangulating the car, suppose the scenario presented in **Fig.2**, where we want to find all the points where the distance to Tower 4 (T_4) is always 60 cm greater than the distance to Tower 1 (T_1) in a city that is 100 cm by 100 cm. The distance difference between the towers is $2a$ according to the hyperbola formula above. Furthermore, placing two towers at the foci of the hyperbola assigns the value of $2c$ to be the distance separating the towers.

B. Two hyperbolas

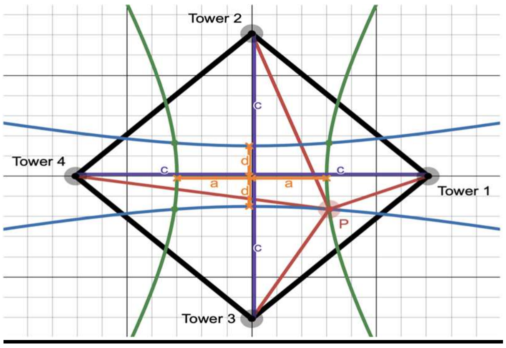

Fig.3. Visualisation of two hyperbolas in the context of locating a Smart Car within the Smart City, as well as point P, which is the only possible location of the Smart Car if given that it is 60cm closer to T_1 than T_4 and is 30 cm closer to T_3 than T_2.

To find the exact position of an object in the city based on the difference in distance from the towers, it is sufficient to draw another hyperbola corresponding to the distance difference between the object and Towers 2 (T_2) and 3 (T_3) and select the intersection that corresponds to the desired point, as illustrated in **Fig.3**. To distinguish this hyperbola from the one representing the T_1 and T_4 axis, the parameter *d* will represent the distance difference between the object and T_2 and T_3 (equivalent to the parameter *a* for the hyperbola of the T_1 and T_4 axis).

C. Intersection of the two hyperbolas

To find the intersection between two hyperbolas, we must first isolate *x* and *y* in the hyperbola equation to assert that $x_1 = x_2$ and $y_1 = y_2$. **Eq.2 & 3** are the transformed equations of the hyperbolas and **Eq.4 & 5** represent the possible intersection points between the hyperbolas presented in **Eq.2 & 3**.

$$\text{Hyperbola 1}: \ x = \pm\sqrt{a^2\left(\left(\frac{y}{b}\right)^2+1\right)}$$

$$\text{Hyperbola 2}: \ y = \pm\sqrt{d^2\left(\left(\frac{x}{f}\right)^2+1\right)}$$

Where

P : Point representing the position of the Smart Car

T_1, T_2, T_3, T_4 : Positions of the Smart Towers 1, 2, 3 and 4.

L : Distance between two adjacent Smart Towers (in a square formation)

c : Diagonal length of the Smart Tower square formation ($\sqrt{2L^2}$)

$$a = \frac{d(P,T_4) - d(P,T_1)}{2} \qquad b = \sqrt{c^2 - a^2}$$

$$d = \frac{d(P,T_3) - d(P,T_2)}{2} \qquad f = \sqrt{c^2 - d^2}$$

Eq.2 & 3. Hyperbolas 1 and 2 representing the possible locations of the Smart Car

$$x = \pm\,|f|\sqrt{\frac{-a^2(b^2+d^2)}{a^2d^2 - b^2f^2}}$$

$$y = \pm\sqrt{\frac{b^2d^2(a^2+f^2)}{b^2f^2 - a^2d^2}}$$

Parameters same as Eq. 2 & 3.

Eq.4 & 5. Points P(*x*,*y*) form the 4 possible results of the intersection of the two hyperbolas (refer to **Fig.3** and **Eq.2 & 3**). By properly interpreting the ± sign, the only real solution is revealed.

D. Interpretation of the ± sign

In these equations, due to the square root, the values of *x* and *y* will always be positive, which will always represent a coordinate in the first quadrant. Therefore, we need to interpret the values of *a* and *d* to decide the sign of *x* and *y*. If *a* is negative, it means that the car is closer to T_4 than T_1, so the *x* coordinate must be negative. Similarly, if *d* is negative, then the car is closer to T_3 than T_2, so the *y* coordinate must be negative.

E. Rotation

Finally, we need to rotate the intersection coordinate obtained to align with the standard Cartesian coordinate system of the city, with the corners of the city located at (L/2, L/2), (L/2,−L/2), (−L/2,−L/2), and (−L/2, L/2), where *L* is the side length of the square city.

To do this, simply multiply the point by the standard rotation matrix with an angle of 45° or *π*/4 radians, which involves performing the rotation matrix calculations on *x* and *y* as shown in **Eq. 6 & 7.**

$$x' = x \cdot \cos(\tfrac{\pi}{4}) - y \cdot \sin(\tfrac{\pi}{4})$$

$$y' = x \cdot \sin(\tfrac{\pi}{4}) + y \cdot \cos(\tfrac{\pi}{4})$$

Eq.6 & 7. Forty-five-degree ($\frac{\pi}{4}$ rad) rotation equations derived from the standard rotation matrix.

F. Height of the towers

Given that the towers and the car are not on the same plane, it is important to consider the height of the towers relative to the ultrasonic module on the car, as the distance traveled by the ultrasound wave is not only horizontal but also slightly angled upwards. A simple adjustment using the Pythagorean theorem can account for this height, as the distance traveled by the ultrasound corresponds to the hypotenuse of a right triangle as illustrated in **Fig.4**, where its height is the difference in height between the towers and the car, and its base is the actual horizontal distance between the wave and the tower.

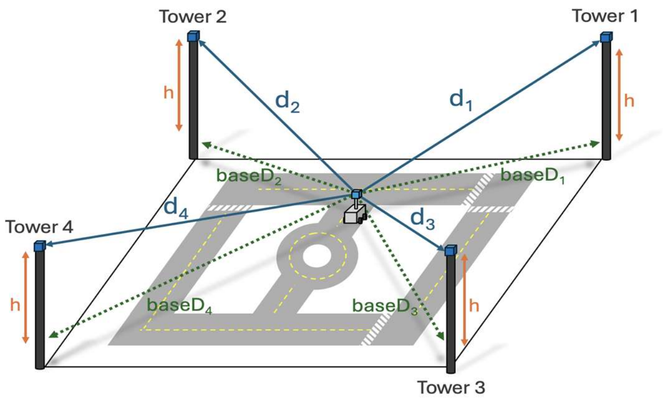

Fig.4. The initial distance measured by the towers (*d*) is not the real horizontal distance (*baseD*) separating them from the car. To find the real distance, the height difference (*h*) between the car transmitter and the tower receiver must be considered.

The formula for converting the distances, **Eq. 8**, is as follows:

$$baseD = \sqrt{d^2 - h^2}$$

Where

baseD is the horizontal distance between the car's antenna and the receiving towers.

d is the distance measured by the towers (based on the time intervals).

h is the height difference between the receiver and the transmitter.

Eq.8. Equation to convert the distance measured by the towers to the actual horizontal distance between the cars and the towers by considering the height of the towers (refer to **Fig.4**)

To account for the height of the towers, the triangulation must be performed twice. The first triangulation calculates the distances measured by the receiving towers, *m*. Subsequently, a second triangulation is performed using the value of *d* obtained from the formula above to find the actual positions of the car and towers.

Here's a step-by-step outline of the process:

1. **Initial Triangulation**: Use the differences in time of ultrasound signal receptions to calculate the measured distances *d* between the car and the towers.
2. **Adjust for Height *h***: Convert the measured distances *d* to horizontal distances *baseD* using **Eq. 8**.
3. **Second Triangulation**: Perform triangulation again using the horizontal distances *baseD* to determine the accurate position of the car.

In summary, the proposed LPS takes advantage of the accessibility of modified ultrasonic HC-SR04 sensors, and the elegance of a hyperbola-based triangulation method to simply, efficiently and accurately determine the position of a Smart Car within the new LRIMa Smart City.

IV. CITY CENTRALIZED WEB INTERFACE: CCWI

The City Centralized Web Interface (CCWI) (see Fig. 5) serves as the central hub for all simulations and real-time monitoring within our LRIMa City system. The interface facilitates efficient management of fog computing on-device solutions, enabling real-time visualization of vehicle locations and other critical data. For storing data and enabling real-time communication between the LPS and CCWI, we leveraged Aliot, an IoT development kit introduced in our previous work [7], that uses a custom protocol based on WebSockets [15] and inspired by MQTT [16].

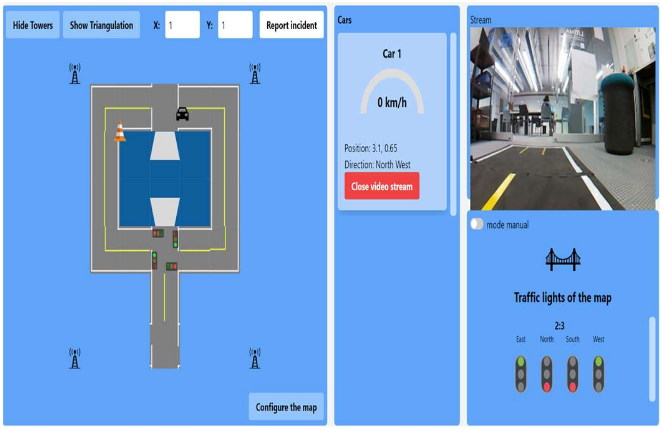

Fig.5. The City Centralized Web Interface (CCWI)

By integrating various elements into a single platform, CCWI enhances the operational efficiency and research capabilities of the LRIMa City (**Fig.6**), making it an indispensable tool for both monitoring, control and simulation.

Fig.6. The LRIMa City

A. Features of CCWI

Multiple features are implemented in CCWI, including customization options for city layout enabling users to replicate their physical Smart City.

Among the notable features is the "Report Incident" function, which broadcasts the location of incidents throughout the LRIMa City. Moreover, CCWI can control various IoT components such as the bridge and traffic lights. Finally, for monitoring LRIMa City's cars, a streaming feature is available, providing real-time visuals from the cars' perspective, along with its position, direction, and speed.

Finally, the CCWI offers a mathematical visualisation of the LPS, illustrating an overlay of the hyperbolas used to triangulate the car's position illustrated in **Fig.7**.

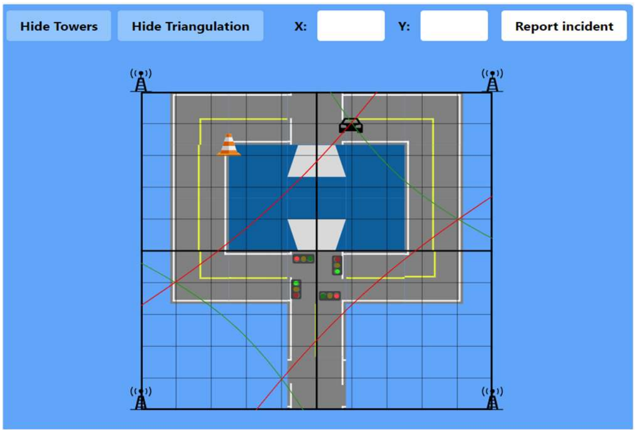

Fig.7. Simulation of the LPS in CCWI.

B. Car Conception

One of the main issues faced when expanding the City was the lack of reusability, upgradability, and standardization of our cars. New features had to be modified to adapt to the different models, which was very time-consuming and prone to inconsistencies. The solution was to develop a standardized car that could be built easily and rapidly. This standardization involved creating a uniform design and using interchangeable parts to ensure compatibility across the entire fleet. The most important components of the new model are the custom printed circuit board (PCB) mounted on a Raspberry Pi model 4 B and the rover-like ultrasonic transmitter tower at the top of the car used in the LPS. The structure being mainly comprised of easily assembled 3D parts and electronic components all listed in [17] accompanied by a comprehensive conception guide accessible at [18], makes building a car from scratch a task that can be completed in less than two hours, which is a significant improvement when compared to the previous fleet of cars.

V. LPS RESULTS

To test the LPS' accuracy, a modified HC-SR04 sensor (transmitter and receiver replaced by wider angle of detection or emission equivalents) acting as the transmitter, *T*, was placed in various locations aiming towards two other modified HC-SR04 sensors, acting as receivers R_1 and R_2 as illustrated in **Fig.8**. By measuring the distance between each receiver and the transmitter, d_1 and d_2, we can find the distance difference Δd between the receivers and compare it to the distance difference calculated via the ultrasounds of the LPS shown on the Serial Plotter of the Arduino IDE used for the tests.

Twelve different receiver and transmitter placements were tested and compiled in **Table. I** to compare the two methods of calculating the distance difference. **Fig.9** illustrates the distance difference measured manually and via the LPS side by side. **Fig.10** illustrates the deviation of the difference illustrated in **Fig.9** and offers a visual representation of the sensors that were at a significant angle in the testing.

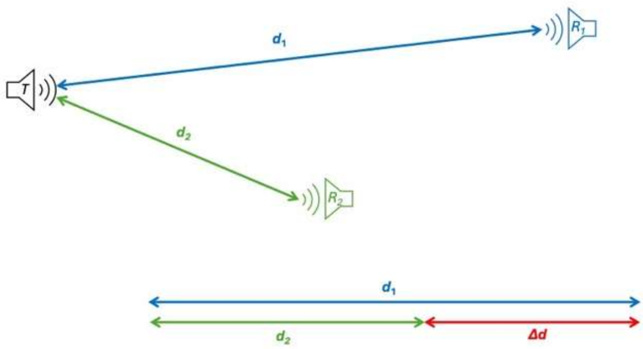

Fig.8. Illustration of the preliminary testing setup that compares manual measurements and LPS calculations

Distance d_1 between R1 and transmitter (±0.5cm)	Distance d_2 between R2 and transmitter (±0.5cm)	Manual measuring difference Δd_m between d_1 and d_2 $\lvert d_2 - d_1 \rvert$(±1cm)	Average time gap measured with LPS system between received signals by R1 and R2 (in µs)	Distance difference Δd_{lps} between d_1 and d_2 measured by LPS (time in µs converted to s • 343 m/s (speed of sound))	Were the sensors at an angle with the transmitters?
145.5	148	**2.5**	90	**3.087**	No
118.5	148	**29.5**	870	**29.841**	No
98	148	**50**	1470	**50.421**	No
77.5	148	**70.5**	2040	**69.972**	No
196	148	**48**	1350	**46.305**	No
117.5	148	**30.5**	890	**30.527**	Yes
183.5	148	**35.5**	1050	**36.015**	Yes
176	148	**28**	820	**28.126**	Yes
148.5	148	**0.5**	25	**0.8575**	Yes
175	148	**27**	730	**25.039**	Yes
192.5	148	**44.5**	1260	**43.218**	Yes
202	148	**54**	1550	**53.165**	Yes

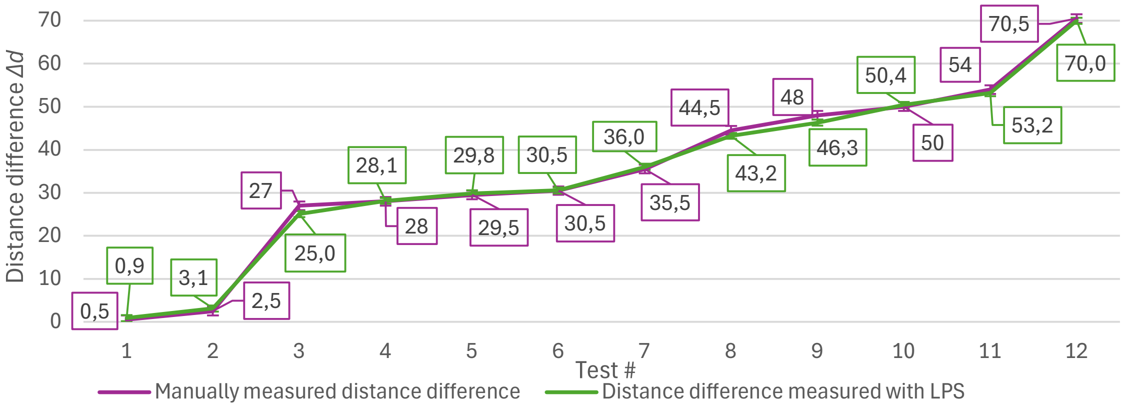

Fig.9. Comparison between the manual distance difference and LPS calculation of the distance difference between the two ultrasonic receivers

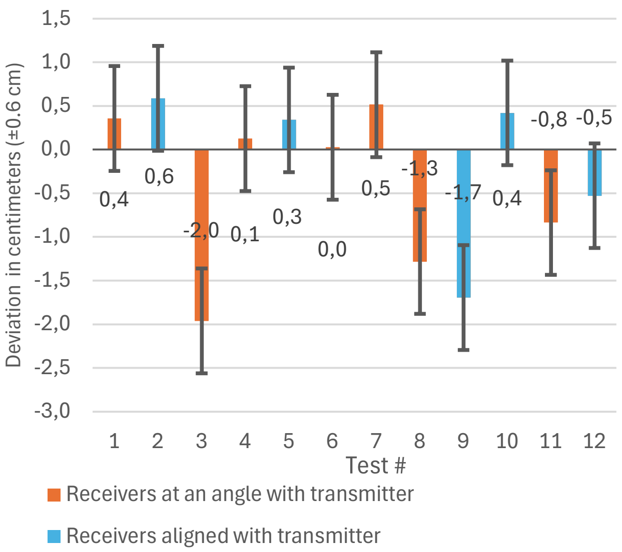

Fig.10. Deviation of LPS calculations compared to manual measuring during testing

As observed in **Fig.10**, the deviation of the LPS compared to manual measuring is within ±2.6cm, representing a very accurate positioning for the needs of the LRIMa Smart City. Sources of error generating these deviations may come from the large quantities of noise being filtered out when measuring the position and the need for an average to properly estimate the distance difference. Additionally, the ultrasonic sensors have undergone modification, potentially decreasing their accuracy.

VI. CONCLUSION

LRIMa City represents an evolving Smart City project that has undergone two distinct development phases. The initial phase focused on implementing IoT solutions through a cloud-based architecture, creating a foundation for a wide array of connected urban systems. In this current phase, the emphasis has shifted to exploring a LPS that should precisely determine the location of a Smart Car within the City and relay its coordinates to a graphical web interface for accurate, real-time visualization. The LPS would exploit the emission of ultrasonic signals by the car and their detection by a network of towers strategically placed at City corners. By measuring the time differences between the receipt of these signals, we can triangulate the vehicle's position using hyperbolas, which is then displayed on the interface to monitor and manage urban traffic effectively.

The LPS would represent a significant advancement in our Smart City framework, complementing a variety of other intelligent components managed through the web interface, including smart motorized vehicles, adaptive streetlights, a speed radar, smart parking solutions, and a remotely controlled bridge.

ACKNOWLEDGMENT

We would like to thank FRQNT and Innovations ALIVEcode inc. for financially supporting this research. We would also like to thank John Molas and Matéo Tardy who greatly contributed to the *Smart City* project as well as Gabriel Risk with his help on the LPS.

REFERENCES

[1] The Smart City feature video: https://www.youtube.com/watch?v=WLPV0lk77A0. [last visited 2 July 2024].

[2] J. Rezgui, E. Soldevila and A. Mokraoui, “LRIMa city: a Fog-Computing-Based Smart City with Integrated Self-Driving Cars”, accepted IEEE ISNCC 2023, Qatar.

[3] Gueuning, F.E.; Varlan, M.; Eugne, C.E.; Dupuis, P. Accurate distance measurement by an autonomous ultrasonic system combining time-of-flight and phase-shift methods. In Proceedings of the Instrumentation and Measurement Technology Conference, Brussels, Belgium, 4–6 June 1996.

[4] Khyam, M.O.; Ge, S.S.; Li, X.; Pickering, M.R. Highly Accurate Time-of-Flight Measurement Technique Based

on Phase-Correlation for Ultrasonic Ranging. IEEE Sens. J. 2017.

[5] Kang-Wook, K.; Jihye, K.; Chang-Gun, L.; Junghee, H. Accurate Indoor Location Tracking Exploiting Ultrasonic Reflections. IEEE Sens. J. 2016, 16, 9075–9088. https://ieeexplore.ieee.org/abstract/document/7590064

[6] A. Yazici, U. Yayan and H. Yücel, "An ultrasonic based indoor positioning system," 2011 International Symposium on Innovations in Intelligent Systems and Applications, Istanbul, Turkey, 2011, pp. 585-589, https://ieeexplore.ieee.org/document/5946154

[7] J. Rezgui and E. Soldevila, "Novel IoT Development Kit for Personalized Smart Ecosystems: Aliot," 2022 IEEE Global Conference on Artificial Intelligence and Internet of Things (GCAIoT), Alamein New City, Egypt, 2022, pp. 54-59.

[8] Ultrasonic distance sensor - 5V (HC-SR04) https://www.sparkfun.com/products/15569

[9] Takagi J, Kanazawa H, Ichikawa K, Mitamura H (2022) A simple intuitive method for seeking intersections of hyperbolas for acoustic positioning biotelemetry. PLoS ONE 17(11): e0276289. https://doi.org/10.1371/journal.pone.0276289

[10] O'Keefe, B. Finding Location with Time of Arrival and Time Difference of Arrival Techniques. ECE

[11] L. Paull et al., "Duckietown: An open, inexpensive and flexible platform for autonomy education and research," 2017 IEEE International Conference on Robotics and Automation (ICRA), Singapore, 2017, pp. 1497-1504, doi: 10.1109/ICRA.2017.7989179.

[12] Clarinval, A.; Simonofski, A.; Henry, J.; Vanderose, B.; Dumas, B. Introducing the Smart City to Children: Lessons Learned from Hands-On Workshops in Classes. Sustainability 2023, 15, 1774. https://doi.org/10.3390/su15031774

[13] Ruiz Vicente, F.; Zapatera Llinares, A.; Montés Sánchez, N. "Sustainable City": A Steam Project Using Robotics to Bring the City of the Future to Primary Education Students. Sustainability 2020, 12, 9696. https://doi.org/10.3390/su12229696

[14] Senior Capstone Project, 2017 Tech Notes, 2017. https://sites.tufts.edu/eeseniordesignhandbook/files/2017/05/FireBrick_OKeefe_F1.pdf

[15] RFC6455, The Websocket Protocol: https://datatracker.ietf.org/doc/html/rfc6455. [last visited 2 July 2024].

[16] Message Queuing Telemetry Transport (MQTT). http://mqtt.org/. [last visited 2 July 2024].

[17] Listing of all the necessary components for the car: https://docs.google.com/spreadsheets/d/1sNicvbYGCIe1v_OG2mG8b3XvKGpfj8nQ/edit?usp=sharing&ouid=116040123030023664324&rtpof=true&sd=true. [last visited 2 July 2024].

[18] Conception guide for the car: https://alivecode.ca/module/d9e938c7-0b25-4652-972b-e3ceecabdc23/view?act=15304. [last visited 2 July 2024].